# pyepal

Release v0.7.0

Kevin Maik Jablonka, Brian Yoo, Berend Smit

# **CONTENTS**

1	Cont	ents		3				
	1.1	1.1 Getting Started						
		1.1.1	Installation	3				
		1.1.2	Which class do I use?	3				
		1.1.3	Running an active learning experiment					
		1.1.4	Caveats and tricks with Gaussian processes	7				
	1.2	Backgr	ound	ç				
		1.2.1	The intuition behind the algorithm	10				
		1.2.2	How do the hyperparameters influence the algorithm?	10				
	1.3	Tutoria	ls	12				
		1.3.1	1. One active learning step using GPR models built with GPy	12				
		1.3.2	2. Active learning with "measure" function and sklearn models	13				
		1.3.3	3. Quantile regression	13				
		1.3.4	4. Plotting a learning curve	14				
	1.4	The Py	ePAL API reference	14				
		1.4.1	The PAL package	14				
		1.4.2	The models package	28				
	1.5	Develo	per notes	29				
		1.5.1	Contribution Guidelines	29				
		1.5.2	Implementing a new PAL class	30				
2	Indic	es and t	ables	33				
Ру	thon I	Module 1	Index	35				
In	dex			37				

This package implements an active learning approach to efficiently and confidently identify the Pareto front with any regression model that can output a mean and a standard deviation.

It works with any number of objectives, missing data, and is highly customizable.

If you find this code useful for your work, please cite:

- Jablonka, K. M.; Giriprasad, M. J.; Wang, S.; Smit, B.; Yoo, B. Bias Free Multiobjective Active Learning for Materials Design and Discovery, ChemRxiv 2020 (10.26434/chemrxiv.13200197.v1).
- Zuluaga, M.; Krause, A.; Püschel, M. E-PAL: An Active Learning Approach to the Multi-Objective Optimization Problem. Journal of Machine Learning Research 2016, 17 (104), 1–32.

CONTENTS 1

2 CONTENTS

**CHAPTER** 

ONE

# **CONTENTS**

# 1.1 Getting Started

# 1.1.1 Installation

We recommend installing PyePAL in a dedicated virtual environment or conda environment. Note that we currently support Python 3.7 and 3.8.

To install the latest stable release use

```
pip install pyepal
```

or the conda channel (recommended)

```
conda install pyepal -c conda-forge
```

The latest version of PyePAL can be installed from GitHub using

```
pip install git+https://github.com/kjappelbaum/pyepal.git
```

On macOS you might need to install libomp (e.g., brew install libomp) for multihreading in some models.

If you want to limit how many CPUs openblas uses, you can export OPENBLAS\_NUM\_THREADS=1

# 1.1.2 Which class do I use?

- For Gaussian processes built with sklearn use PALSklearn
- For Gaussian processes built with GPy use PALGPy
- For coregionalized Gaussian processes (built with GPy) use PALCoregionalized
- For quantile regression using LightGBM gradient boosted decision trees use PALGBDT
- For infinite wide neural networks with the neural tangent kernel or exact Bayesian inference (Novak et al., 2019) use PALNT
- For an ensemble of finite width neural networks (Lakshminarayanan et al., 2017) (built with JAX) use PALNTEnsemble

If your favorite model is not listed, you can easily implement it yourself (see Implementing a new PAL class)!

# 1.1.3 Running an active learning experiment

The examples directory contains a Jupyter notebook with an example that can also be run on MyBinder.

If using a Gaussian process model built with sklearn or GPy we recommend using a pre-built class such as PALSklearn, PALCoregionalized, PALGPy and following the subsequent steps (for more details on which class to use see Which class do I use?):

- 1. For each objective create a model (if using a coregionalized Gaussian process model, only one model needs to be created)
- 2. Sample a few initial points from the design space. We provide the get\_kmeans\_samples() utilities that can help with the sampling. Our code assumes that X is a np. array.

3. Now we can initialize the instance of one PAL class. If using a sklearn Gaussian process model, we would use

```
from pyepal import PALSklearn

# Each of these models is an instance of sklearn.gaussian_process.
    GaussianProcessRegressor
models = [gpr0, gpr1, gpr2]

# We always need to provide the feature matrix (X), a list of models, and the number of objectives
palinstance = PALSklearn(X, models, 3)

# Now, we can also feed in the first measurements
# this here assumes that we have all measurements for y and we now
# provide those which are present in the indices array
palinstance.update_train_set(indices, y[indices])

# Now we can run one step
next_idx = palinstance.run_one_step()
```

At this level, we have a range of different optional arguements we can set.

- epsilon: one  $\epsilon$  per dimension in a np.ndarray. This can be used to set different tolerances for each objective. Note that  $\epsilon_i \in [0,1]$ .
- delta: the  $\delta$  hyperparameter ( $\delta \in [0,1]$ ). Increasing this value will speed up the convergence.
- beta\_scale: an empirical scaling parameter for  $\beta$ . The theoretical guarantees in the PAL paper are derived for this parameter set to 1. But in practice, a much faster convergence can be achieved by setting it to a number  $0 < \beta_{\rm scale} \ll 1$ .
- goal: By default, PyePAL assumes that the goal is to maximize every objective. If this is not the case, this argument can be set using a list of "min" and "max" strings, with "min" specifying whether to minimize the ith objective and "max" indicating whether to maximize this objective.

• coef\_var\_threshold: By default, PyePAL will not consider points with a coefficient of variation  $\geq 3$  for the classification step of the algorithm. This is meant to avoid classifying design points for which the model is entirely unsure. This tends to happen when a model is severely overfit on the training data (i.e., the training data uncertainties are very low, whereas the prediction uncertainties are very high). To change this setting, reduce this value to make the check tighter or increase it to avoid this check (as in the original implementation).

In the case of missing observations, i.e., only two of three outputs are measured, report the missing observations as np.nan. The call could look like

for a case in which we performed measurements for samples with index 1 and 2 of our design space, but did not measure the first target for sample 2.

### Hyperparameter optimization

Usually, the hyperparameters of a machine learning model, in particular the kernel hyperparameters of a Gaussian process regression model, should be optimized as new training data is added. However, since this is usually a computationally expensive process, it may not be desirable to perform this at every iteration of the active learning process. The iteration frequency of the hyperparameter optimization is internally set by the \_should\_optimize\_hyperparameters function, which by default uses a schedule that optimizes the hyperparameter every 10th iteration. This behavior can be changed by override this function.

#### Reclassification schedule

A problem of the e-PAL algorithm can be the case where the initial GPR predictions are wrong. This might lead to points that are actually Pareto-efficient being confidently discarded (in case the GPR predicts with low variance a performance that is dominated by some other point). Under the assumption that the GPR predictions improve over the course of the use of the algorithm, this behavior can be mitigated by "resetting" all the classification, i.e. reconsidering discarded points. In PyePAL, we automatically do this in case there is only one point left. In general, you might want to customize this behavior, which you can do, for example, by monkey patching

```
from pyepal.pal.schedules import linear

def my_schedule(self):
    return linear(self.iteration, 1)

PALGPyReclassify._should_reclassify = my_schedule
```

or by subclassing

```
from pyepal.pal.schedules import linear
class PALGPyReclassify(PALGPy):
    def _should_reclassify(self):
        return linear(self.iteration, 1)
```

In the examples above the full design space would be re-classified each iteration.

### Logging

Basic information such as the current iteration and the classification status are logged and can be viewed by printing the PAL object

```
print(palinstance)

# returns: pyepal at iteration 1. 10 Pareto optimal points, 1304 discarded points,

→200 unclassified points.
```

We also provide calculation of the hypervolume enclosed by the Pareto front with the function get hypervolume()

```
hv = get_hypervolume(palinstance.means[palinstance.pareto_optimal])
```

# Properties of the PAL object

For debugging there are some properties and attributes of the *PAL* class that can be used to inspect the progress of the active learning loop.

- get the points in the design space, x:
  - design\_space returns the full design space matrix
  - pareto optimal points: returns the points that are classified as Pareto-efficient
  - sampled\_points: returns the points that have been sampled
  - discarded\_points: returns the points that have been discarded
- get the indices of Pareto efficent, sampled, discarded, and unclassified points with pareto\_optimal\_indices, sampled\_indices, discarded\_indices, and unclassified indices
- similarly, the number of points in the different classes can be obtained using number\_pareto\_optimal\_points, number\_discarded\_points, number\_unclassified\_points, and number\_sampled\_points. The total number of design points can be obtained with number\_design\_points.
- hyperrectangle\_size returns the sizes of the hyperrectangles, i.e., the weights that are used in the sampling step
- means and std contain the predictions of the model
- sampled is a mask array. In case one objective has not been measured its cell is False

#### Exploring a space where all objectives are known

In some cases, we may already posess all measurements, but would like to run PAL with different settings to test how the algorithm performs. In this case, we provide the <code>exhaust\_loop()</code> wrapper.

```
from pyepal import PALSklearn, exhaust_loop
models = [gpr0, gpr1, gpr2]
palinstance = PALSklearn(X, models, 3)
exhaust_loop(palinstance, y)
```

This will continue calling run one step () until there is no unclassified sample left.

# **Batch sampling**

By default, the run\_one\_step function of the PAL classes will return a np.ndarray with only one index for the point in the design space for which the next experiment should be performed. In some situations, it may be more practical to run multiple experiments as batches before running the next active learning iteration. In such cases, we provide the batch\_size argument which can be set to an integer greater than one.

```
next_idx = palinstance.run_one_step(batch_size=10)
# next_idx will be a np.array of length 10
```

Note that the *exhaust\_loop* also supports the *batch\_size* keyword argument

```
palinstance = PALSklearn(X, models, 3)

# sample always 10 points and do this until there is no unclassified
# point left
exhaust_loop(palinstance, y, batch_size=10)
```

### Adding new points to the design matrix

In some applications, you might want to augment the design matrix after a few iterations of PyePAL. This could be useful, for example, if you start with a coarse discretization of your design space then want to refine this grid in subsequent iterations in the relevant regions of the design space.

Adding new points to the design matrix can be easily achieved using the augment\_design\_matrix() function that takes the new design matrix as input. By default, it will run the current model for the new, augmented, design matrix, and re-classify all points. You can turn this behavior off using the clean\_classify parameter.

Alternatively, you can use the classify flag that keeps all previous classifications. This means that if there is a point that was previously Pareto-efficient in the non-augmented design space but is now dominated by a new design point, it will no longer certainly be classified as Pareto-efficient.

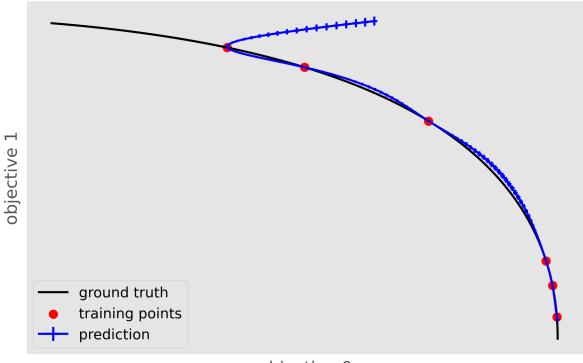
Note that is important that the new points are sampled from the same distribution as the previous points in the design space. Otherwise, the model will have to deal with unexpected data shift.

# Plotting a learning curve

To plot a learning curve one can use the plot\_learning\_curve() function. This can be useful to estimate if the number of initial points is reasonable and to see if the model is predictive.

# 1.1.4 Caveats and tricks with Gaussian processes

One caveat to keep in mind is that  $\epsilon$ -PAL will not work if the predictive variance does not make sense, for example, when the model is overconfident and the uncertainties for the training set is significantly lower than those for the predicted set. In this case, PyePAL will untimely, and often incorrectly, label the design points. An example situation where the predictions for an overconfident model due to a training set that excludes a part of design space is shown in the figure below



objective 0

This problem is exacerbated in conjunction with  $\beta_{\text{scale}} < 1$ . To make the model more robust we suggest trying:

- to set reasonable bounds on the length scale parameters
- to increase the regularization parameter/noise kernel (alpha in sklearn)
- to increase the number of data points, especially the coverage of the design space
- to use a kernel that suits the problem

8

• to turn off ARD. Automatic relevance determination (ARD) might increase the predictive performance, but also makes the model more prone to overfitting

We also recommend cross-validating the Gaussian process models and checking that the predicted variances make sense. When performing cross-validation, make sure that the index provided to PyePAL is the same size as the cross-validation folds. By default, the code will run a simple cross-validation only on the first iteration and provide a warning if the mean absolute error is above the mean standard deviation. The warning will look something like

```
The mean absolute error in cross-validation is 64.29, the mean variance is 0.36. Your model might not be predictive and/or overconfident. In the docs, you find hints on how to make GPRs more robust.
```

This behavior can changed with the cross-validation test being performed more frequently by overriding the should\_run\_crossvalidation function.

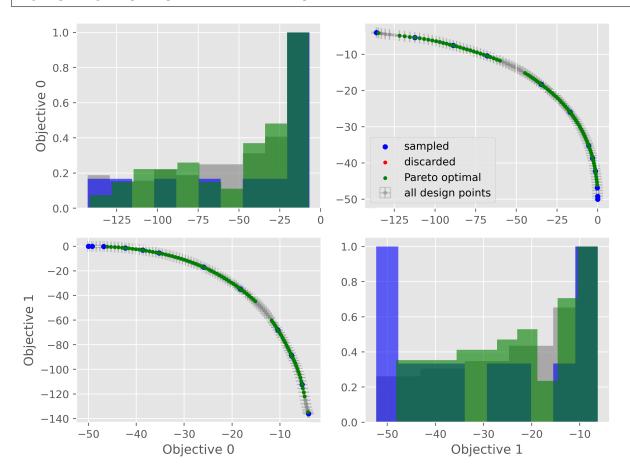
Another way to detect overfitting is to use plot\_jointplot() function from the plotting subpackage. This function will plot all objectives against each other (with errorbars and different classes indicated with colors) and histograms of the objectives on the diagonal. If the majority of predicted points tend to overlap one another and get discarded by PyePAL, this may suggest that the surrogate model is overfitted.

```
from pyepal.plotting import plot_jointplot
```

(continues on next page)

(continued from previous page)

```
# palinstance is a instance of a PAL class after
# calling run_one_step
fig = plot_jointplot(palinstance.means, palinstance)
```



# 1.2 Background

This package implements a modified version of the -PAL algorithm from Zuluaga et al. in an object-oriented design. The algorithm effeciently searches for the Pareto efficient points in an unbiased manner for any number of dimensions. This package can be used with any regression model that can output means and standard deviations.

This implementation has the following features:

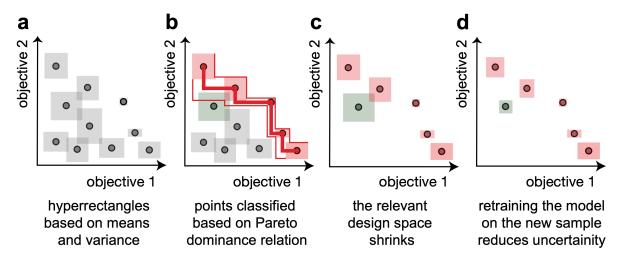
- We ensure that the sampling is scale-invariant and that the algorithm can deal with positive and negative objective values.
- In contrast to the original implementation by Zuluaga et al. we do not assume that the range of the objectives is known a priori. In their implementation it is used to calculate fixed tolerance values  $\epsilon_i \cdot r_i$  (where  $r_i$  is the range of objective i). We instead use by default  $\epsilon_i \cdot |\mu_i|$ .
- Instead of using the predicted  $\hat{\mu}$  and  $\hat{\sigma}$  also for the sampled points we use the measured  $\mu$  and  $\sigma$ .
- This implementation is directly scalable to *n*-dimensional problems.
- It can be easily used with any kind of regression model with uncertainty measures. For example, one can replace the Gaussian process model with a neural network with Dropout Monte Carlo for the uncertainty estimate.

1.2. Background 9

• The support for missing data is implemented. For example, if you measure for labeled data for only some of the objectives, you will simply need to provide *np.nan* for the missing measurements. The code will automatically estimate these measurements. If using coregionalized GPR models, the models will try to utilize the correlations between the objectives to improve these predictions.

In our own work, we used this algorithm for materials discovery applications.

# 1.2.1 The intuition behind the algorithm



The PAL algorithm iterates through the following steps:

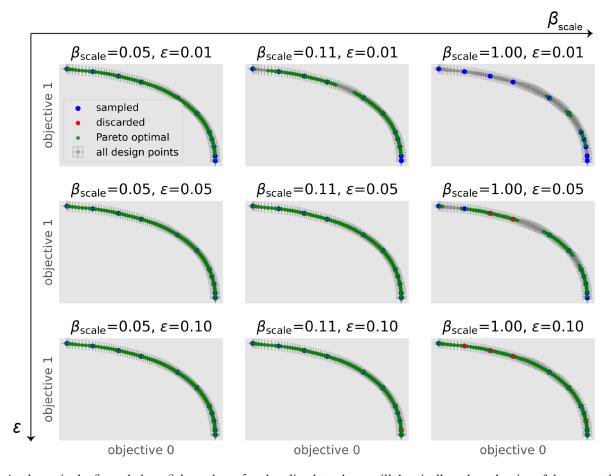
- a. Training a machine learning model to predict means and standard deviations for all points of the design space. This can be used to construct uncertainty hyperrectangles.
- b. Using these points, we can use the Pareto dominance relation to classify points as Pareto optimal or to discard them. In some cases, e.g., when uncertainty hyperrectangles overlap, we will not be able to perform a classification with confidence. This is different from many Bayesian optimization approaches that couples an acquisition function such as expected improvement, which introduces a total order in the design space and hence biases the search.
- c. Since in step (b) we discard many points (with confidence) the effective design space shrinks. This enables us to sample the next experiment from the regions near the Pareto front with points labeled as Pareto optimal and "unclassified". Since the ultimate goal is to perform a classification of the full design space—with confidence—we sample the point with the largest hyperrectangle.
- d. Retraining a model with this new measurement will reduce the width of the hyperrectangles.

# 1.2.2 How do the hyperparameters influence the algorithm?

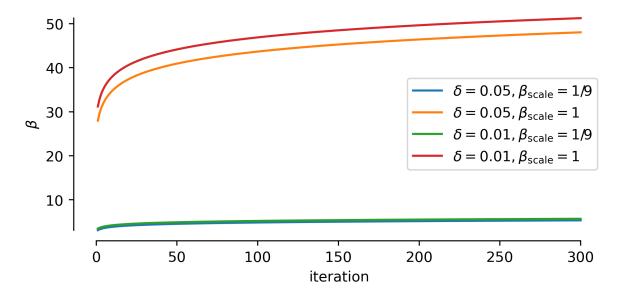
To test the influence of hyperparameter settings we provide an example where we ran one step of the algorithm on the Binh-Korn test function.

We model every objective seperately with a Matérn-3/2 kernel, leave  $\delta = 0.05$  fixed and vary  $\epsilon$  and  $\beta_{\text{scale}}$ .

We find that increasing  $\epsilon$  speeds up the algorithm, but gives us a sparser Pareto frontier. Similarly,  $\beta_{\text{scale}}$  speeds up the algorithm by scaling the size of the hyperrectangles.



As shown in the figure below,  $\beta$  depends on  $\delta$  and scaling beta down will drastically reduce the size of the uncertainty rectangles and in this way influence the convergence behavior.

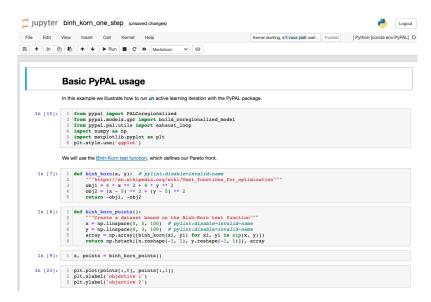


1.2. Background 11

# 1.3 Tutorials

To explore different use cases of PyePAL, we recommend checking out the example notebooks. All notebooks can be run without installation on MyBinder. In the folder you find the notebooks with pre-executed output cells. Rerunning them should take no more than a few minutes.

# 1.3.1 1. One active learning step using GPR models built with GPy

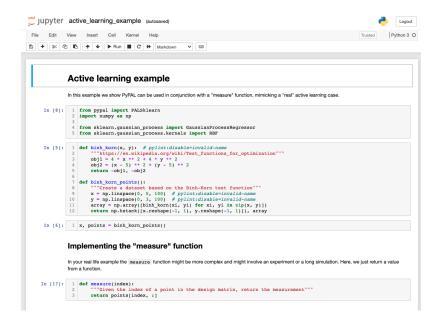


# **Topic covered**

12

- building a pal\_coregionalized GPR model using build\_coregionalized\_model()
- using coregionalized models with PALCoregionalized
- attributes of the PAL instance
- exhaust\_loop()

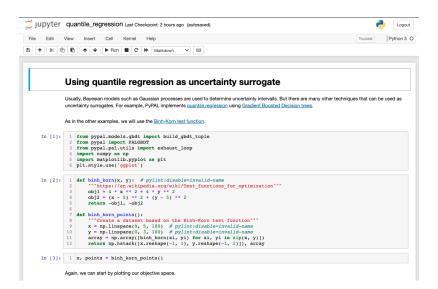
# 1.3.2 2. Active learning with "measure" function and sklearn models



# **Topic covered**

- using sklearn models with PALSklearn
- selecting an initial set with get\_maxmin\_samples()
- plotting with plot\_jointplot()

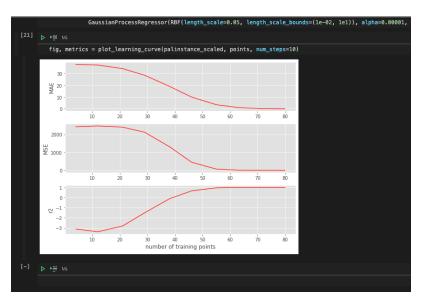
# 1.3.3 3. Quantile regression



- Using LightGBM models with quantile loss with PALGBDT
- selecting an initial set with get\_kmeans\_samples()

1.3. Tutorials

# 1.3.4 4. Plotting a learning curve



- Using sklearn Gaussian process models with RBF kernels
- Plotting learning curves

# 1.4 The PyePAL API reference

# 1.4.1 The PAL package

# **Core functions**

Core functions for PAL

### **Base class**

Base class for PAL

```
class pyepal.pal_base.PALBase (X_design, models, ndim, epsilon=0.01, delta=0.05, beta_scale=0.11111111111111111, goals=None, coef_var_threshold=3, ranges=None)
```

Bases: object

PAL base class

\_\_init\_\_(X\_design, models, ndim, epsilon=0.01, delta=0.05, beta\_scale=0.1111111111111111, goals=None, coef\_var\_threshold=3, ranges=None)
Initialize the PAL instance

- X\_design (np.array) Design space (feature matrix)
- models (list) Machine learning models
- **ndim** (*int*) Number of objectives

- epsilon (Union[list, float], optional) Epsilon hyperparameter. Defaults to 0.01.
- delta (float, optional) Delta hyperparameter. Defaults to 0.05.
- **beta\_scale** (*float*, *optional*) Scaling parameter for beta. If not equal to 1, the theoretical guarantees do not necessarily hold. Also note that the parametrization depends on the kernel type. Defaults to 1/9.
- goals (List[str], optional) If a list, provide "min" for every objective that shall be minimized and "max" for every objective that shall be maximized. Defaults to None, which means that the code maximizes all objectives.
- **coef\_var\_threshold** (*float*, optional) Use only points with a coefficient of variation below this threshold in the classification step. Defaults to 3.
- ranges (np.ndarray, optional) Numpy array of length ndmin, where each element contains the value range of given objective. If this is provided, we will use  $\epsilon \cdot ranges$  to computer the uncertainties of the hyperrectangles instead of the default behavior  $\epsilon \cdot |\mu|$

```
__repr__()
Return repr(self).
weakref
```

list of weak references to the object (if defined)

 $\verb"augment_design_space" (X\_design, classify = False, clean\_classify = True)$ 

Add new design points to PAL instance

#### **Parameters**

- **X\_design** (np.ndarrary) Design matrix. Two-dimensional array containing measurements in the rows and the features as the columns.
- **classify** (bool) Reclassifies the new design space, using the old model. This is, it runs inference, calculates the hyperrectangles, and runs the classification. Does not increase the iteration count. Note though that points that already have been classified as Pareto-optimal will not be re-classified, e.g., discarded—even if the new design points dominate the existing "Pareto optimal" points. Defaults to False.
- clean\_classify (bool) Reclassifies the new design space, using the old model. This is, it runs inference, calculates the hyperrectangles, and runs the classification. Does not increase the iteration count. But, in contrast to classify it erases all previous classifications, before running the new classification. Hence, if some new design point dominates a previously Pareto efficient point, the previous Pareto optimal point will no longer be classified as Pareto efficient. This flag is incompatible with classify. If you choose clean\_classify, PyePAL will erase all previous classifications, independent of what you choose for classify. Defaults to True.

Return type None

#### property discarded\_indices

Return the indices of the discarded points

# property discarded\_points

Return the discarded points

# property hyperrectangle\_sizes

Return the sizes of the hyperrectangles

# property number\_design\_points

Return the number of points in the design space

# property number\_discarded\_points

Return the nnumber of discarded points

## property number\_pareto\_optimal\_points

Return the number of Pareto optimal points

#### property number\_sampled\_points

Return the number of sampled points

#### property number\_unclassified\_points

Return the number of unclassified points

#### property pareto\_optimal\_indices

Return the indices of the Pareto optimal points

#### property pareto\_optimal\_points

Return the pareto optimal points

[summary]

#### **Parameters**

- batch\_size (int, optional) Number of indices that will be returned. Defaults to 1.
- **pooling\_method** (str) Method that is used to aggregate the uncertainty in different objectives into one scalar. Available options are: "fro" (Frobenius/Euclidean norm), "mean", "median". Defaults to "fro".
- **sample\_discarded** (bool) if true, it will sample from all points and not only from the unclassified and Pareto optimal ones
- use\_coef\_var (bool) If True, uses the coefficient of variation instead of the unscaled rectangle sizes
- replace\_mean (bool) If true uses the measured means for the sampled points
- replace\_std (bool) If true uses the measured standard deviation for the sampled points

Raises ValueError - In case the PAL instance was not initialized with measurements.

#### Returns

Returns array of indices if there are unclassified points that can be sample left.

**Return type** Union[np.array, None]

**sample** (exclude\_idx=None, pooling\_method='fro', sample\_discarded=False, use\_coef\_var=True)

Runs the sampling step based on the size of the hyperrectangle. I.e., favoring exploration.

- **exclude\_idx** (*Union[np.array, None], optional*) Points in design space to exclude from sampling. Defaults to None.
- **pooling\_method** (str) Method that is used to aggregate the uncertainty in different objectives into one scalar. Available options are: "fro" (Frobenius/Euclidean norm), "mean", "median". Defaults to "fro".

- sample\_discarded (bool) if true, it will sample from all points and not only from the unclassified and Pareto optimal ones
- use\_coef\_var (bool) If True, uses the coefficient of variation instead of the unscaled rectangle sizes

**Raises ValueError** – In case there are no uncertainty rectangles, i.e., when the \_predict has not been successfully called.

**Returns** Index of next point to evaluate in design space

Return type int

#### property sampled\_indices

Return the indices of the sampled points

### property sampled\_mask

Create a mask for the sampled points We count a point as sampled if at least one objective has been measured, i.e., self.sampled is a N \* number objectives array in which some columns can be false if a measurement has not been performed

#### property sampled points

Return the sampled points

#### should\_cross\_validate()

Override for more complex cross validation schedules

#### property unclassified\_indices

Return the indices of the unclassified points

#### property unclassified\_points

Return the discarded points

#### update\_train\_set (indices, measurements, measurement\_uncertainty=None)

Update training set following a measurement

#### **Parameters**

- indices (np.ndarray) Indices of design space at which the measurements were taken
- measurements (np.ndarray) Measured values, 2D array. the length must equal the length of the indices array. the second direction must equal the number of objectives. If an objective is missing, provide np.nan. For example, np.array([1, 1, np.nan])
- measurement\_uncertainty (np.ndarray) uncertainty in the measuremens, if not provided (None) will be zero. If it is not None, it must be an array with the same shape as the measurements If an objective is missing, provide np.nan. For example, np.array([1, 1, np.nan])

#### property uses\_fixed\_epsilon

True if it uses the fixed epsilon  $\epsilon \cdot ranges$ 

### For GPy models

```
PAL using GPy GPR models
```

#### **Parameters**

- X\_design (np.array) Design space (feature matrix)
- models (list) Machine learning models
- **ndim** (*int*) Number of objectives
- **epsilon** (*Union*[*list*, *float*], *optional*) Epsilon hyperparameter. Defaults to 0.01.
- **delta** (float, optional) Delta hyperparameter. Defaults to 0.05.
- **beta\_scale**(float, optional) Scaling parameter for beta. If not equal to 1, the theoretical guarantees do not necessarily hold. Also note that the parametrization depends on the kernel type. Defaults to 1/9.
- **goals** (List[str], optional) If a list, provide "min" for every objective that shall be minimized and "max" for every objective that shall be maximized. Defaults to None, which means that the code maximizes all objectives.
- **coef\_var\_threshold** (*float*, *optional*) Use only points with a coefficient of variation below this threshold in the classification step. Defaults to 3.
- **restarts** (*int*) Number of random restarts that are used for hyperparameter optimization. Defaults to 20.
- n\_jobs (int) Number of parallel processes that are used to fit the GPR models. Defaults to 1.

# For coregionalized GPy models

PAL for coregionalized GPR models

- **X\_design** (np.array) Design space (feature matrix)
- models (list) Machine learning models
- ndim (int) Number of objectives
- epsilon (Union[list, float], optional) Epsilon hyperparameter. Defaults to 0.01.

- **delta** (float, optional) Delta hyperparameter. Defaults to 0.05.
- **beta\_scale** (*float*, *optional*) Scaling parameter for beta. If not equal to 1, the theoretical guarantees do not necessarily hold. Also note that the parametrization depends on the kernel type. Defaults to 1/9.
- goals (List[str], optional) If a list, provide "min" for every objective that shall be minimized and "max" for every objective that shall be maximized. Defaults to None, which means that the code maximizes all objectives.
- **coef\_var\_threshold** (*float*, optional) Use only points with a coefficient of variation below this threshold in the classification step. Defaults to 3.
- **restarts** (*int*) Number of random restarts that are used for hyperparameter optimization. Defaults to 20.
- parallel (bool) If true, model hyperparameters are optimized in parallel, using the GPy implementation. Defaults to False.

#### For sklearn GPR models

PAL using Sklearn GPR models

```
class pyepal.pal_pal_sklearn.PALSklearn(*args, **kwargs)
    Bases: pyepal.pal_base.PALBase
```

PAL class for a list of Sklearn (GPR) models, with one model per objective

```
__init__ (*args, **kwargs)

Construct the PALSklearn instance
```

- X design (np.array) Design space (feature matrix)
- models (list) Machine learning models. You can provide a list of GaussianProcess-Regressor instances or a list of *fitted* RandomizedSearchCV/GridSearchCV instances with GaussianProcessRegressor models
- **ndim** (*int*) Number of objectives
- **epsilon** (*Union[list, float], optional*) Epsilon hyperparameter. Defaults to 0.01.
- **delta** (float, optional) Delta hyperparameter. Defaults to 0.05.
- **beta\_scale** (*float*, *optional*) Scaling parameter for beta. If not equal to 1, the theoretical guarantees do not necessarily hold. Also note that the parametrization depends on the kernel type. Defaults to 1/9.
- **goals** (List[str], optional) If a list, provide "min" for every objective that shall be minimized and "max" for every objective that shall be maximized. Defaults to None, which means that the code maximizes all objectives.
- **coef\_var\_threshold** (*float*, optional) Use only points with a coefficient of variation below this threshold in the classification step. Defaults to 3.
- **n\_jobs** (*int*) Number of parallel processes that are used to fit the GPR models. Defaults to 1.

### For quantile regression with LightGBM

Implements a PAL class for GBDT models which can predict uncertainty intervals when used with quantile loss. For an example of GBDT with quantile loss see Jablonka, Kevin Maik; Moosavi, Seyed Mohamad; Asgari, Mehrdad; Ireland, Christopher; Patiny, Luc; Smit, Berend (2020): A Data-Driven Perspective on the Colours of Metal-Organic Frameworks. ChemRxiv. Preprint. https://doi.org/10.26434/chemrxiv.13033217.v1

For general information about quantile regression see https://en.wikipedia.org/wiki/Quantile\_regression

Note that the scaling of the hyperrectangles has been derived for GPR models (with RBF kernels).

```
class pyepal.pal.pal_gbdt.PALGBDT(*args, **kwargs)
    Bases: pyepal.pal.pal_base.PALBase

PAL class for a list of LightGBM GBDT models
__init__(*args, **kwargs)
    Construct the PALGBDT instance
```

- X\_design (np.array) Design space (feature matrix)
- (List[Iterable[LGBMRegressor (models) Machine learning models. You need to provide a list of iterables. One iterable per objective and every iterable contains three LGBMRegressors. The first one for the lower uncertainty limits, the middle one for the median and the last one for the upper limit. To create appropriate models, you need to use the quantile loss. If you want to parallelize training, we recommend that you use the LightGBM parallelization and fit the models for the different objectives in serial fashion.s
- LGBMRegressor Machine learning models. You need to provide a list of iterables. One iterable per objective and every iterable contains three LGBMRegressors. The first one for the lower uncertainty limits, the middle one for the median and the last one for the upper limit. To create appropriate models, you need to use the quantile loss. If you want to parallelize training, we recommend that you use the LightGBM parallelization and fit the models for the different objectives in serial fashion.s
- LGBMRegressor]] Machine learning models. You need to provide a list of iterables. One iterable per objective and every iterable contains three LGBMRegressors. The first one for the lower uncertainty limits, the middle one for the median and the last one for the upper limit. To create appropriate models, you need to use the quantile loss. If you want to parallelize training, we recommend that you use the LightGBM parallelization and fit the models for the different objectives in serial fashion.s
- ndim (int) Number of objectives
- **epsilon** (*Union[list, float], optional*) Epsilon hyperparameter. Defaults to 0.01.
- **delta** (float, optional) Delta hyperparameter. Defaults to 0.05.
- **beta\_scale** (*float*, *optional*) Scaling parameter for beta. If not equal to 1, the theoretical guarantees do not necessarily hold. Also note that the parametrization depends on the kernel type. Defaults to 1/9.
- **goals** (List[str], optional) If a list, provide "min" for every objective that shall be minimized and "max" for every objective that shall be maximized. Defaults to None, which means that the code maximizes all objectives.
- **coef\_var\_threshold** (*float*, optional) Use only points with a coefficient of variation below this threshold in the classification step. Defaults to 3.

• interquartile\_scaler (float, optional) – Used to convert the difference between the upper and lower quantile into a standard deviation. This, is std = (uplow)/interquartile\_scaler. Defaults to 1.35, following Wan, X., Wang, W., Liu, J. et al. Estimating the sample mean and standard deviation from the sample size, median, range and/or interquartile range. BMC Med Res Methodol 14, 135 (2014). https://doi.org/10.1186/1471-2288-14-135

#### For GPR with GPFlow

PAL using GPy GPR models

```
class pyepal.pal_gpflowgpr.PALGPflowGPR(*args, **kwargs)
    Bases: pyepal.pal_base.PALBase
```

PAL class for a list of GPFlow GPR models, with one model per objective. Please consider that there are specific multioutput models (https://gpflow.readthedocs.io/en/master/notebooks/advanced/multioutput.html) for which the train and prediction function would need to be adjusted. You might also consider using streaming GPRs (https://github.com/thangbui/streaming\_sparse\_gp). In future releases we might support this case automatically (i.e., handle the case in which only one model is provided).

```
__init__ (*args, **kwargs)
Contruct the PALGPflowGPR instance
```

- X\_design (np.array) Design space (feature matrix)
- models (list) Machine learning models
- **ndim** (*int*) Number of objectives
- **epsilon** (*Union[list, float], optional*) Epsilon hyperparameter. Defaults to 0.01.
- delta (float, optional) Delta hyperparameter. Defaults to 0.05.
- **beta\_scale** (float, optional) Scaling parameter for beta. If not equal to 1, the theoretical guarantees do not necessarily hold. Also note that the parametrization depends on the kernel type. Defaults to 1/9.
- **goals** (List[str], optional) If a list, provide "min" for every objective that shall be minimized and "max" for every objective that shall be maximized. Defaults to None, which means that the code maximizes all objectives.
- **coef\_var\_threshold** (*float*, optional) Use only points with a coefficient of variation below this threshold in the classification step. Defaults to 3.
- **opt** (function, optional) Optimizer function for the GPR parameters. If None (default), then we will use `gpflow.optimizers.Scipy()`
- opt\_kwargs (dict, optional) Keyword arguments passed to the optimizer. If None, PyePAL will pass ["maxiter": 100]
- n\_jobs (int) Number of parallel threads that are used to fit the GPR models. Defaults to 1.

# Schedules for hyperparameter optimization

Provides some scheduling functions that can be used to implement the \_should\_optimize\_hyperparameters function

```
pyepal.pal.schedules.exp_decay (iteration, base=10)
```

Optimize hyperparameters at logartihmically spaced intervals

#### **Parameters**

- iteration (int) current iteration
- base (int, optional) Base of the logarithm. Defaults to 10.

Returns True if iteration is on the log scaled grid

Return type bool

```
pyepal.pal.schedules.linear(iteration, frequency=10)
```

Optimize hyperparameters at equally spaced intervals

#### **Parameters**

- iteration (int) current iteration
- frequency (int, optional) Spacing between the True outputs. Defaults to 10.

Returns True if iteration can be divided by frequency without remainder

Return type bool

# Utilities for multiobjective optimization

Utilities for dealing with Pareto fronts in general

```
pyepal.pal.utils.dominance_check (point1, point2)
```

One point dominates another if it is not worse in all objectives and strictly better in at least one. This here assumes we want to maximize

```
Return type bool
```

```
pyepal.pal.utils.dominance_check_jitted(point, array)
```

Check if point dominates any point in array

```
Return type bool
```

```
pyepal.pal.utils.dominance_check_jitted_2 (array, point)
```

Check if any point in array dominates point

```
Return type bool
```

```
pyepal.pal.utils.dominance_check_jitted_3 (array, point, ignore_me)
```

Check if any point in array dominates point. ignore\_me since numba does not understand masked arrays

```
Return type bool
```

```
pyepal.pal.utils.exhaust_loop(palinstance, y, batch_size=1)
```

Helper function that takes an initialized PAL instance and loops the sampling until there is no unclassified point left. This is useful if all measurements are already taken and one wants to test the algorithm with different hyperparameters.

#### **Parameters**

• palinstance (PALBase) – A initialized instance of a class that inherited from PALBase and implemented the .\_train() and .\_predict() functions

- **y** (np.array) Measurements. The number of measurements must equal the number of points in the design space.
- batch\_size (int, optional) Number of indices that will be returned. Defaults to 10.

Returns None. The PAL instance is updated in place

```
pyepal.pal.utils.get_hypervolume (pareto_front, reference_vector, prefactor=-1)
```

Compute the hypervolume indicator of a Pareto front I multiply it with minus one as we assume that we want to maximize all objective and then we calculate the area

```
f1 | |----| | -| | -| f2
```

But the code we use for the hv indicator assumes that the reference vector is larger than all the points in the Pareto front. For this reason, we then flip all the signs using prefactor

This indicator is not needed for the epsilon-PAL algorithm itself but only to allow tracking a metric that might help the user to see if the algorithm converges.

```
Return type float
```

```
pyepal.pal.utils.get_kmeans_samples (X, n_samples, **kwargs)
```

Get the samples that are closest to the k=n samples centroids

#### **Parameters**

- **X** (np.array) Feature array, on which the KMeans clustering is run
- n\_samples (int) number of samples are should be selected
- passed to the KMeans(\*\*kwargs)-

**Returns** selected\_indices

Return type np.array

Greedy maxmin sampling, also known as Kennard-Stone sampling (1). Note that a greedy sampling is not guaranteed to give the ideal solution and the output will depend on the random initialization (if this is chosen).

If you need a good solution, you can restart this algorithm multiple times with random initialization and different random seeds and use a coverage metric to quantify how well the space is covered. Some metrics are described in (2). In contrast to the code provided with (2) and (3) we do not consider the feature importance for the selection as this is typically not known beforehand.

You might want to standardize your data before applying this sampling function.

Some more sampling options are provided in our structure\_comp (4) Python package. Also, this implementation here is quite memory hungry.

References: (1) Kennard, R. W.; Stone, L. A. Computer Aided Design of Experiments. Technometrics 1969, 11 (1), 137–148. https://doi.org/10.1080/00401706.1969.10490666. (2) Moosavi, S. M.; Nandy, A.; Jablonka, K. M.; Ongari, D.; Janet, J. P.; Boyd, P. G.; Lee, Y.; Smit, B.; Kulik, H. J. Understanding the Diversity of the Metal-Organic Framework Ecosystem. Nature Communications 2020, 11 (1), 4068. https://doi.org/10.1038/s41467-020-17755-8. (3) Moosavi, S. M.; Chidambaram, A.; Talirz, L.; Haranczyk, M.; Stylianou, K. C.; Smit, B. Capturing Chemical Intuition in Synthesis of Metal-Organic Frameworks. Nat Commun 2019, 10 (1), 539. https://doi.org/10.1038/s41467-019-08483-9. (4) https://github.com/kjappelbaum/structure\_comp

#### **Parameters**

• **X** (np.array) – Feature array, this is the array that is used to perform the sampling

- $n_{samples}(int)$  number of points that will be selected, needs to be lower than the length of X
- metric (str, optional) Distance metric to use for the maxmin calculation. Must be a valid option of scipy.spatial.distance.cdist ('braycurtis', 'canberra', 'chebyshev', 'cityblock', 'correlation', 'cosine', 'dice', 'euclidean', 'hamming', 'jaccard', 'jensenshannon', 'kulsinski', 'mahalanobis', 'matching', 'minkowski', 'rogerstanimoto', 'russellrao', 'seuclidean', 'sokalmichener', 'sokalsneath', 'sqeuclidean', 'wminkowski', 'yule'). Defaults to 'euclidean'
- init (str, optional) either 'mean', 'median', or 'random'. Determines how the initial point is chosen. Defaults to 'center'
- **seed** (*int*, optional) seed for the random number generator. Defaults to None.
- passed to the cdist(\*\*kwargs)-

Returns selected\_indices

Return type np.array

pyepal.pal.utils.is pareto efficient (costs, return mask=True)

Find the Pareto efficient points Based on https://stackoverflow.com/questions/ 32791911/fast-calculation-of-pareto-front-in-python

#### **Parameters**

- costs (np.array) An (n\_points, n\_costs) array
- return\_mask (bool, optional) True to return a mask, Otherwise it will be a (n efficient points,) integer array of indices. Defaults to True.

**Returns** [description]

Return type np.array

# **Utilities for plotting**

### Plotting utilities

Plot stacked barplots for every step of the iteration.

#### **Parameters**

- pareto\_optimal (np.ndarray) Number of pareto optimal points for every iteration.
- non\_pareto\_points (np.ndarray) Number of discarded points for every iteration
- unclassified\_points (np.ndarray) Number of unclassified points for every iteration

### Returns

matplotlib axis (the same that was provided as input or one from a new figure if no axis was provided)

Return type axis

```
pyepal.plotting.plot_histogram(y, palinstance, ax=None)
```

Plot histograms, with maxima scaled to one and different categories indicated in color for one objective

#### **Parameters**

- **y** (np.ndarray) objective (measurement)
- palinstance (PALBase) instance of a PAL class
- ax (ax) Matplotlib figure axis

#### Returns

**matplotlib axis (the same that was provided as input** or one from a new figure if no axis was provided) *f* 

#### Return type ax

pyepal.plotting.plot\_jointplot (y, palinstance, labels=None, figsize=(8.0, 6.0))

Plot a jointplot of the objective space with histograms on the diagonal and 2D-Pareto plots on the off-diagonal.

#### **Parameters**

- **y** (np.array) Two-dimensional array with the objectives (measurements)
- palinstance (PALBase) "trained" PAL instance
- labels (Union[List[str], None], optional)—Labels for each objective. Defaults to "objective [index]".
- figsize (tuple, optional) Figure size for joint plot. Defaults to (8.0, 6.0).

Returns matplotlib Figure object.

### Return type fig

pyepal.plotting.plot\_pareto\_front\_2d(y\_0, y\_1, std\_0, std\_1, palinstance, ax=None)
Plot a 2D pareto front, with the different categories indicated in color.

# **Parameters**

- y\_0 (np.ndarray) objective 0
- y\_1 (np.ndarray) objective 1
- **std\_0** (np.ndarray) standard deviation objective 0
- **std\_1** (np.ndarray) standard deviation objective 0
- palinstance (PALBase) PAL instance
- ax (axix, optional) Matplotlib figure axis. Defaults to None.

#### Returns

matplotlib axis (the same that was provided as input or one from a new figure if no axis was provided)

# Return type ax

pyepal.plotting.plot\_residuals(y, palinstance, labels=None, figsize=(6.0, 4.0))

Plot signed residual (on y axis) vs fitted (on x axis) plot of sampled points. Will create suplots for y.ndim > 1.

- y (np.array) Two-dimensional array with the objectives (measurements)
- palinstance (PALBase) "trained" PAL instance
- labels (Union[List[str], None], optional)—Labels for each objective. Defaults to "objective [index]".

• **figsize** (tuple, optional) - Figure size for each individual residual vs fitted objective plot. Defaults to (6.0, 4.0).

Returns matplotlib Figure object

Return type fig

# Input validation

```
Methods to validate inputs for the PAL classes
```

```
pyepal.pal.validate_inputs.base_validate_models(models)
```

Currently no validation as the predict and train function are implemented independet of the base class

```
Return type list
```

```
pyepal.pal.validate_inputs.validate_beta_scale (beta_scale)
```

**Parameters** beta\_scale (Any) – scaling factor for beta

**Raises** ValueError – If beta is smaller than 0

**Returns** scaling factor for beta

Return type float

```
pyepal.pal.validate_inputs.validate_coef_var (coef_var)
```

Make sure that the coef var makes sense

```
pyepal.pal.validate_inputs.validate_coregionalized_gpy (models)
```

Make sure that model is a coregionalized GPR model

```
pyepal.pal.validate_inputs.validate_delta(delta)
```

Make sure that delta is in a reasonable range

Parameters delta (Any) – Delta hyperparameter

**Raises** ValueError – Delta must be in [0,1].

Returns delta

Return type float

```
pyepal.pal.validate_inputs.validate_epsilon(epsilon, ndim)
```

Validate epsilon and return a np.array

#### **Parameters**

- **epsilon** (Any) Epsilon hyperparameter
- ndim (int) Number of dimensions/objectives

#### Raises

- **ValueError** If epsilon is a list there must be one float per dimension
- ValueError Epsilon must be in [0,1]
- ValueError If epsilon is an array there must be one float per dimension

Returns Array of one epsilon per objective

Return type np.ndarray

```
pyepal.pal.validate_inputs.validate_gbdt_models(models, ndim)
```

Make sure that the number of iterables is equal to the number of objectives and that every iterable contains three LGBMRegressors. Also, we check that at least the first and last models use quantile loss

```
Return type List[Iterable]
```

pyepal.pal.validate\_inputs.validate\_goals(goals, ndim)

Create a valid array of goals. 1 for maximization, -1 for objectives that are to be minimized.

#### **Parameters**

- goals (Any) List of goals, typically provideded as strings 'max' for maximization and 'min' for minimization
- **ndim** (*int*) number of dimensions

#### Raises

- ValueError If goals is a list and the length is not equal to ndim
- ValueError If goals is a list and the elements are not strings 'min', 'max' or -1 and 1

**Returns** Array of -1 and 1

Return type np.ndarray

```
pyepal.pal.validate_inputs.validate_gpy_model(models)
```

Make sure that all elements of the list a GPRegression models

pyepal.pal.validate\_inputs.validate\_interquartile\_scaler (interquartile\_scaler)

Make sure that the interquartile\_scaler makes sense

Return type float

pyepal.pal.validate\_inputs.validate\_ndim(ndim)

Make sure that the number of dimensions makes sense

Parameters ndim (Any) – number of dimensions

#### Raises

- ValueError If the number of dimensions is not an integer
- ValueError If the number of dimensions is not greater than 0

**Returns** the number of dimensions

Return type int

```
pyepal.pal.validate_inputs.validate_njobs(njobs)
```

Make sure that njobs is an int > 1

Return type int

```
pyepal.pal.validate_inputs.validate_nt_models(models, ndim)
```

Make sure that we can work with a sequence of pyepal.pal.models.nt.NTModel()

Return type Sequence

```
pyepal.pal.validate_inputs.validate_number_models(models, ndim)
```

Make sure that there are as many models as objectives

#### **Parameters**

- models (Any) List of models
- **ndim** (*int*) Number of objectives

Raises ValueError - If the number of models does not equal the number of objectives

of the variance

28

Return type Tuple[array, array]

```
pyepal.pal.validate_inputs.validate_optimizers(optimizers, ndim)
     Make sure that we can work with a Sequence if JaxOptimizer
          Return type Sequence
pyepal.pal.validate_inputs.validate_positive_integer_list (seq,
                                                                                ndim, parame-
                                                                          ter name='Parameter')
     Can be used, e.g., to validate and standardize the ensemble size and epochs input
          Return type Sequence[int]
pyepal.pal.validate_inputs.validate_sklearn_gpr_models(models, ndim)
     Make sure that there is a list of GPR models, one model per objective
          Return type List[GaussianProcessRegressor]
1.4.2 The models package
Helper functions for GPR with GPy
Wrappers for Gaussian Process Regression models.
We typically use the GPy package as it offers most flexibility for Gaussian processes in Python. Typically, we use
automatic relevance determination (ARD), where one lengthscale parameter per input dimension is used.
If your task requires training on larger training sets, you might consider replacing the models with their sparse version
but for the epsilon-PAL algorithm this typically shouldn't be needed.
For kernel selection, you can have a look at https://www.cs.toronto.edu/~duvenaud/cookbook/ Matérn, RBF and Ra-
tionalQuadrat are good quick and dirty solutions but have their caveats
pyepal.models.gpr.build_coregionalized_model(X_train, y_train, kernel=None, w_rank=1,
                                                          **kwargs)
     Wrapper for building a coregionalized GPR, it will have as many outputs as y_train.shape[1]. Each output will
     have its own noise term
          Return type GPCoregionalizedRegression
pyepal.models.gpr.build model(X train, y train, index=0, kernel=None, **kwargs)
     Build a single-output GPR model
          Return type GPRegression
pyepal.models.gpr.get_matern_32_kernel(NFEAT, ARD=True, **kwargs)
     Matern-3/2 kernel without ARD
          Return type Matern32
pyepal.models.gpr.get_matern_52_kernel(NFEAT, ARD=True, **kwargs)
     Matern-5/2 kernel without ARD
          Return type Matern52
pyepal.models.gpr.get_ratquad_kernel(NFEAT, ARD=True, **kwargs)
     Rational quadratic kernel without ARD
          Return type RatQuad
pyepal.models.gpr.predict (model, X)
```

Wrapper function for the prediction method of a GPy regression model. It return the standard deviation instead

Chapter 1. Contents

```
pyepal.models.gpr.predict_coregionalized(model, X, index=0)
```

Wrapper function for the prediction method of a coregionalized GPy regression model. It return the standard deviation instead of the variance

```
Return type Tuple[array, array]
```

```
pyepal.models.gpr.set_xy_coregionalized(model, X, y, mask=None)
```

Wrapper to update a coregionalized model with new data

# 1.5 Developer notes

# 1.5.1 Contribution Guidelines

# **Commit messages**

• To automatically generate the changelog and releases we use conventional commits use the prefix feat for new features, chore for updating grunt tasks etc; no production code change, fix for bug fixes and docs for changes to the documentation. Use feat!:, or fix!:, refactor!:, etc., to represent a breaking change (indicated by the !). This will result in bump of the SemVer major version number.

# Python code

Please install the pre-commit hooks using

to automatically

- format the code with black
- sort the imports with isort
- lint the code with pylint

We use type hints, which we feel is a good way of documentation and helps us find bugs using mypy.

Some of the pre-commit hooks modify the files, e.g., they trim whitespaces or format the code. If they modify your file, you will have to run git add and git commit again. To skip the pre-commit checks (not recommended) you can use git commit --no-verify.

#### **New features**

Please make a new branch for the development of new features. Rebase on the upstream master and include a test for your new feature. (The CI checks for a drop in code coverage.)

#### Releases

Releases are automated using a GitHub actions based on the commit message. Maintainers manually upload the release to PyPi.

# 1.5.2 Implementing a new PAL class

If you want to use PyePAL with a model that we do not support yet, i.e., not GPy or sklearn Gaussian process regression, it is easy to write your own class. For this, you will need to inherit from PALBase and implement your \_train and \_predict() functions (and maybe also the pyepal.pal.pal\_base.PALBase.\_set\_hyperparameters and pyepal.pal.pal\_base.PALBase.\_should\_optimize\_hyperparameters functions) using the design\_space and y attributes of the class.

For instance, if we develop some multioutput model that has a train() and a predict() method, we could simply use the following design pattern

```
from pyepal import PALBase

class PALMyModel(PALBase):
    def _train(self):
        self.models[0].train(self.design_space[self.sampled], self.y[self.sampled])

def _predict(self):
        self.mu, self.std = self.models[0].predict(self.design_space)
```

Note that we typically provide the models, even if it is only one, in a list to keep the API consistent.

In some instances, you may want to perform an operation in parallel, e.g., train the models for different objectives in parallel. One convenient way to do this in Python is by using concurrent.futures. The only caveat to this that this approach requires that the function is picklable. To ensure this, you may want to implement the function that you want to parallelize, outside the class. For example, you could use the following design pattern

```
from pyepal import PALBase
import concurrent.futures
from functools import partial
def _train_model_picklable(i, models, design_space, objectives, sampled):
   model = models[i]
   model.fit(
        design_space[sampled[:, i]],
        objectives[sampled[:, i], i].reshape(-1, 1),
    return model
class MyPal (PALBase) :
    def __init__(self, *args, **kwargs):
       n_jobs = kwargs.pop("n_jobs", 1)
        validate_njobs(n_jobs)
        self.n_{jobs} = n_{jobs}
        super().__init__(*args, **kwargs)
        validate_number_models(self.models, self.ndim)
    def _train(self):
        train_single_partial = partial(
            _train_model_picklable,
            models=self.models,
            design_space=self.design_space,
            objectives=self.y,
            sampled=self.sampled,
        models = []
```

(continues on next page)

(continued from previous page)

```
with concurrent.futures.ProcessPoolExecutor(
    max_workers=self.n_jobs
) as executor:
    for model in executor.map(train_single_partial, range(self.ndim)):
        models.append(model)
self.models = models
```

# **CHAPTER**

# TWO

# **INDICES AND TABLES**

- genindex
- modindex
- search

# **PYTHON MODULE INDEX**

# p

```
pyepal.models.gpr, 28
pyepal.pal.core, 14
pyepal.pal.pal_base, 14
pyepal.pal.pal_coregionalized, 18
pyepal.pal.pal_gbdt, 20
pyepal.pal.pal_gpflowgpr, 21
pyepal.pal.pal_gpy, 18
pyepal.pal.pal_sklearn, 19
pyepal.pal.schedules, 22
pyepal.pal.utils, 22
pyepal.pal.validate_inputs, 26
pyepal.plotting, 24
```

36 Python Module Index

# **INDEX**

Symbols	dominance_check_jitted_3() (in module		
init() (pyepal.pal_base.PALBase method),	pyepal.pal.utils), 22		
init() (pyepal.pal.pal_coregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALCoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALcoregionalized.PALco	exhaust_loop() (in module pyepal.pal.utils), 22 exp_decay() (in module pyepal.pal.schedules), 22		
init() (pyepal.pal.pal_gpflowgpr.PALGPflowGPR	G		
method), 21init() (pyepal.pal_gpy.PALGPy method), 18init() (pyepal.pal_pal_sklearn.PALSklearn	<pre>get_hypervolume() (in module pyepal.pal.utils), 23 get_kmeans_samples() (in module</pre>		
method), 19	get_matern_32_kernel() (in module		
repr() (pyepal.pal_base.PALBase method),  15weakref (pyepal.pal.pal_base.PALBase at-	<pre>pyepal.models.gpr), 28 get_matern_52_kernel()</pre>		
tribute), 15	<pre>get_maxmin_samples() (in module</pre>		
A augment_design_space()	<pre>get_ratquad_kernel() (in module           pyepal.models.gpr), 28</pre>		
(pyepal.pal.pal_base.PALBase method), 15	Н		
_	hyperrectangle_sizes()		
B base_validate_models() (in module	(pyepal.pal_base.PALBase property), 15		
base_validate_models() (in module	(pyepal.pal.pal_base.PALBase property),		
base_validate_models() (in module pyepal.pal.validate_inputs), 26 build_coregionalized_model() (in module pyepal.models.gpr), 28	<pre>(pyepal.pal_base.PALBase property), 15    is_pareto_efficient() (in module</pre>		
base_validate_models() (in module pyepal.pal.validate_inputs), 26 build_coregionalized_model() (in module pyepal.models.gpr), 28 build_model() (in module pyepal.models.gpr), 28	<pre>(pyepal.pal_base.PALBase property), 15    is_pareto_efficient() (in module</pre>		
base_validate_models() (in module pyepal.pal.validate_inputs), 26 build_coregionalized_model() (in module pyepal.models.gpr), 28 build_model() (in module pyepal.models.gpr), 28  D	<pre>(pyepal.pal_base.PALBase property), 15    is_pareto_efficient() (in module</pre>		

<pre>pyepal.pal.schedules, 22 pyepal.pal.utils, 22 pyepal.pal.validate_inputs, 26</pre>	<pre>pyepal.pal.gbdt   module, 20 pyepal.pal_gpflowgpr</pre>		
pyepal.plotting,24	module, 21 pyepal.pal.pal_gpy		
	module, 18 pyepal.pal_sklearn		
<pre>number_design_points()</pre>	module, 19 pyepal.pal.schedules		
number_discarded_points()	module, 22		
(pyepal.pal_base.PALBase property), 16	pyepal.pal.utils module, 22		
number_pareto_optimal_points() (pyepal.pal.pal_base.PALBase property), 16	<pre>pyepal.pal.validate_inputs     module, 26 pyepal.plotting</pre>		
number_sampled_points()	module, 24		
(pyepal.pal_base.PALBase property), 16	R		
number_unclassified_points()	run_one_step() (pyepal.pal_base.PALBase method), 16		
16	S		
P	sample() (pyepal.pal.pal_base.PALBase method), 16		
PALBase (class in pyepal.pal.pal_base), 14	sample() (pyepal.pal.pal.pal.pal_base.PALBase memoa), 10 sampled_indices() (pyepal.pal.pal_base.PALBase		
PALCoregionalized (class in	property), 17		
pyepal.pal_coregionalized), 18 PALGBDT (class in pyepal.pal_gbdt), 20	sampled_mask() (pyepal.pal_base.PALBase property), 17		
PALGPflowGPR (class in pyepal.pal.pal_gpflowgpr), 21 PALGPy (class in pyepal.pal_gpy), 18	<pre>sampled_points() (pyepal.pal_base.PALBase</pre>		
PALSklearn (class in pyepal.pal.pal_sklearn), 19 pareto_optimal_indices()	<pre>set_xy_coregionalized() (in module</pre>		
(pyepal.pal_base.PALBase property), 16	should_cross_validate() (pyepal.pal.pal_base.PALBase method),		
<pre>pareto_optimal_points()</pre>	17 <b>U</b>		
<pre>plot_bar_iterations() (in module</pre>	unclassified_indices()		
<pre>pyepal.plotting), 24 plot_histogram() (in module pyepal.plotting), 24</pre>	(pyepal.pal.pal_base.PALBase property), 17		
plot_jointplot() (in module pyepal.plotting), 25	unclassified_points()		
plot_pareto_front_2d() (in module pyepal.plotting), 25	(pyepal.pal.pal_base.PALBase property), 17		
plot_residuals() (in module pyepal.plotting), 25	<pre>update_train_set()</pre>		
<pre>predict() (in module pyepal.models.gpr), 28 predict_coregionalized() (in module</pre>	(pyepal.pal_base.PALBase method),		
pyepal.models.gpr), 28	17 uses_fixed_epsilon()		
pyepal.models.gpr module, 28	(pyepal.pal.pal_base.PALBase property), 17		
pyepal.pal.core module,14	V		
pyepal.pal.pal_base module,14	<pre>validate_beta_scale() (in module</pre>		
pyepal.pal_coregionalized	<pre>pyepal.pal.validate_inputs), 26 validate_coef_var() (in module</pre>		
module, 18	pyepal.pal.validate_inputs), 26		

38 Index

validate_coregionaliz		(in module
pyepal.pal.validate_i	nputs), 26	
validate_delta()	(in	module
pyepal.pal.validate_i	nputs), 26	
<pre>validate_epsilon()</pre>	(in	module
pyepal.pal.validate_i	nputs), 26	
<pre>validate_gbdt_models(</pre>	) (in	module
pyepal.pal.validate_i	nputs), 26	
validate_goals()	(in	module
pyepal.pal.validate_i	nputs), 27	
<pre>validate_gpy_model()</pre>	(in	module
pyepal.pal.validate_i	nputs), 27	
validate_interquartil	e_scaler	() (in mod-
ule pyepal.pal.valida	te_inputs), 27	7
<pre>validate_ndim()</pre>	(in	module
pyepal.pal.validate_i	nputs), 27	
validate_njobs()	(in	module
pyepal.pal.validate_i	nputs), 27	
<pre>validate_nt_models()</pre>	(in	module
pyepal.pal.validate_i	nputs), 27	
validate_number_model	s() (in	module
pyepal.pal.validate_i		
<pre>validate_optimizers()</pre>	(in	module
pyepal.pal.validate_i	nputs), 27	
validate_positive_int	eger_list	() (in mod-
ule pyepal.pal.valida	te_inputs), 28	3
validate_sklearn_gpr_	models()	(in module
pyepal.pal.validate_i		
<u> </u>	- '	

Index 39